

## MATHEMATICAL MODELS OF RELIABILITY OF POWER SUPPLY SYSTEMS

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**Abstract.** Ensuring reliability is one of the most important issues in the creation and operation of any technical system. This is especially important for complex systems, such as power supply systems with internal and external connections consisting of many elements. The task of ensuring the reliability of power supply systems includes a whole set of technical, economic and organizational measures aimed at reducing damage caused by violations of the normal mode of operation of electricity consumers. To obtain information about the reliability of elements of power supply systems, it is necessary to study a fairly large number of identical elements over a long period of time. In the real conditions of an industrial enterprise, this creates certain difficulties, so it is possible to conduct an accelerated reliability test using mathematical modeling [3].

The article discusses some ways to solve the main problem of reliability theory - building a mathematical model and obtaining criteria for selecting optimal options for power supply systems. In this paper, the main materials for calculating the reliability of power supply systems using the mathematical apparatus of probability theory are presented.

**Keywords:** reliability, power supply, mathematical model, parameters, system redundancy, failure, probability theory.

Currently, the electric power industry faces urgent tasks, one of the main ones is to ensure the reliability of the power system. It is necessary to pay attention to the quality of electricity supply and take into account that the requirements for the electric power system are increasing every year.

Mathematical models represent a quantitative relationship between reliability criteria (target functions) and other important factors (independent variables) that characterize the process with real accuracy.

Two types of models are used to calculate reliability indicators - the reliability model and the failure model. The simplest are failure models, which are mathematical descriptions of the failure process.

In power supply systems, non-recoverable elements can be considered low-voltage power devices, and the cost of their repair is equal to the cost of a new device. First of all, let's consider a simple situation in which a sudden failure of the device is possible. It can only be in two states – working and non-working. The transition from a working state to a non-working State is characterized by the intensity of  $\lambda$  failure, and the reverse transition is not possible [1].

The probability of non-stop operation in the transition process and the probability of failure are described by the following differential equations:

$$\frac{dP}{dt} = -\lambda P(t) \qquad \frac{dQ}{dt} = \lambda P(t), \qquad (1)$$

where  $P(t)$  - probability of non-stop operation,  
 $Q(t)$  – probability of failure.

For initial conditions  $P(0) = 1$  and  $Q(0) = 0$  the solution of differential equations has the form:

$$P(t) = \exp(-\lambda t) \qquad Q(t) = [1 - \exp(-\lambda t)] \qquad (2)$$

Hence, knowing that according to the reference data, the failure rate  $\lambda = 0,0005$  1/year, we determine the probability of failure-

free operation and the probability of failure of an electromagnetic contactor with a voltage of (2) 0.4 kV for the time  $t = 6$  years.

$$P(t) = \exp(-\lambda t) = \exp(-0,0005 \cdot 6) = 0,997;$$

$$Q(t) = 1 - \exp(-\lambda t) = 1 - \exp(-0,0005 \cdot 6) = 0,003.$$

If the element is dominated by gradual failures, they can be represented as a set of States  $E_1, E_2...E_N$ , in which the system consistently stays as it wears out, and in  $N$  stages the element finally wears out and failure occurs [1].

The instantaneous parameter of the flow of impacts that wear the element by  $1/N$  part is assumed to be equal to  $\lambda_1$ :

$$\lambda_1 = \lim_{t \rightarrow 0} \frac{P_1(t, t + \Delta t)}{\Delta t} \qquad (3)$$

where  $P_1(t, t + \Delta t)$  - probability of one transition.

In addition to gradual wear, the system may also experience sudden random failures with an intensity of  $\lambda_0$ . As a result, the

differential transition equations for each  $k$ -th state have the form:

$$\frac{dP_k}{dt} = -(\lambda_0 + \lambda_1)P_k(t) + \lambda_1 P_{k-1}(t); \qquad (4)$$

$$\frac{dP_N}{dt} = -(\lambda_0 + \lambda_1)P_N(t);$$

If we consider the system performance as the sum of the States  $E_1, E_2... E_N$  then the probability of failure-free operation is equal to:

$$P(t) = \sum_{k=1}^N P_k(t), \qquad (5)$$

$$\frac{dP}{dt} = \sum_{k=1}^N \frac{dP_k}{dt} = -\lambda_1 P_1(t) - \lambda_0 \sum_{k=1}^N P_k(t) = -\lambda_1 P_1(t) - \lambda_0 P(t) \qquad (6)$$

**Reliability models for restored elements.** Let's consider a more complex case when elements of the power supply system are subject to restoration. At the same time, their reliability depends on the speed of

recovery and the multiplicity of backup elements.

In the simplest case, an element without redundancy can be in two States:  $E_1$  – functional,  $E_0$  – inoperable. If the failure

flow parameter of the system being restored is equal to  $\omega$ , a recovery rate is  $\mu$  ( $\mu = 1/\tau$ )

Differential equations with respect to transition probabilities have the form:

$$\begin{aligned} \frac{dP_1}{dt} &= -\omega P_1(t) + \mu P_0(t); \\ \frac{dP_0}{dt} &= \omega P_1(t) - \mu P_0(t); \end{aligned} \quad (7)$$

Where  $P_1(t)$  - probability of finding the installation in the  $E_1$  state

$P_0(t)$  - probability of finding the installation in the  $E_0$ .

Under the initial conditions  $P_1(0) = 1$ ,  $P_0(0) = 0$  and taking into account that the States  $E_1$  and  $E_0$  Represent a complete group of events, i.e.  $P_0(t) + P_1(t) = 1$ , the solution of differential equations (7) will take the form:

$$P_1(t) = \frac{\mu}{\omega + \mu} \left( 1 + \frac{\omega}{\mu} \cdot \exp[-t(\omega + \mu)] \right); \quad (8)$$

$$P_0(t) = \frac{\mu}{\omega + \mu} (1 - \exp[-t(\omega + \mu)]). \quad (9)$$

Let us consider some special cases that follow from expressions (8) and (9):

a) for instantaneous automatic recovery ( $\omega / \mu = 0$ )  $P_1(t) = 1$ , i.e. the element is absolutely functional at any arbitrary time;

b) if there is no recovery ( $\omega / \mu = \infty$ ):

$$P_1(t) = \exp(\omega t) = \exp(-\lambda t),$$

that is, the probability of state  $E_1$  is equal to the probability of failure-free operation;

c) for a sufficiently large  $t$  ( $t \rightarrow \infty$ ) the probability  $P_1(t)$  (ceases to depend on time (figure 1):

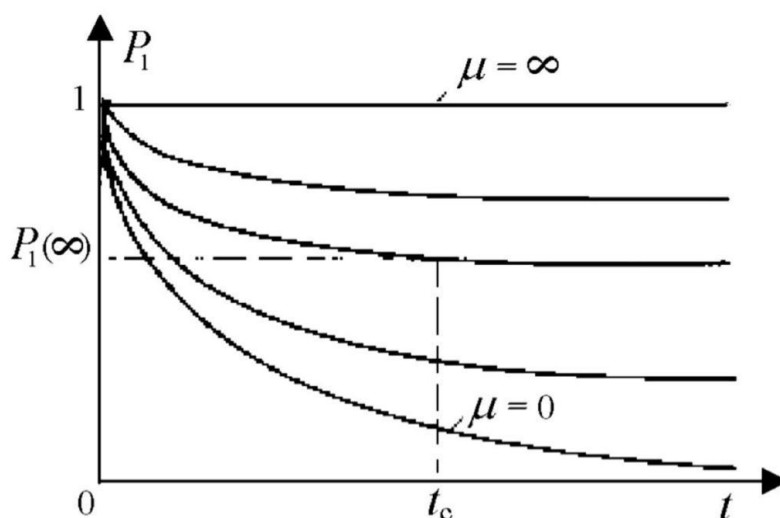


Figure 1 - The dependence of the probability of a healthy state on time at different recovery rates  
Рисунок 1 - Зависимость вероятности здорового состояния от времени при различных скоростях восстановления

$$P_1(\infty) = \frac{\mu}{\omega + \mu} = \frac{t^P}{t^P + t^{\Pi}} = k_{\Gamma} \quad (10)$$

As can be seen from the expression (10) the value  $P_1(\infty)$  is an estimate of the readiness coefficient. Therefore, if there is no backup, recovery increases reliability only in terms of availability, but the probability of uptime does not increase [1].

**Reliability models for redundant systems.** Consider reliability models for a set of interrelated elements. The simplest scheme

from the point of view of reliability theory is a set of elements in which the failure of one element causes the failure of the entire system, but does not change the reliability of other elements (Figure 2). In reliability theory, such a structure is called a sequential connection of elements.

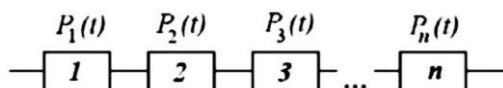


Figure 2 - Block diagram of the reliability of a system with a serial connection of elements  
Рисунок 2 - Структурная схема надежности системы с последовательным соединением элементов

Using the laws of probability theory for complex random events, we obtain that the probability of failure-free operation of such a

system is defined as the probability of failure-free operation of all elements during time  $t$ :

$$P(t) = P_1(t) \cdot P_2(t) \cdot \dots \cdot P_n(t) = \prod_{i=1}^n P_i(t); \quad (11)$$

where  $P_i(t)$  – probability of failure-free operation of the  $i$ -th element.

By expressing  $P_i(t)$  in terms of the bounce rate parameter, we get:

$$P(t) = \exp\left[-\int_0^t \omega(x) dx\right] = \prod_{i=1}^n \exp\left[-\int_0^t \omega_i(x) dx\right] = \exp\left[-\sum_{i=1}^n \int_0^t \omega_i(x) dx\right], \quad (12)$$

$$\omega(t) = \sum_{i=1}^n \omega_i(t) \quad (13)$$

The structure of series-connected elements can be used to model the reliability of electrical circuits with a series connection of electrical devices, transformers, cables and overhead lines, as well as secondary control circuits containing relay windings and

contacts, resistors, thyristors, inductors and electronic devices.

A structure with a parallel connection of elements (figure 3) simulates such a system of  $n$  elements or pieces of equipment, in which  $r$  elements are required for normal operation, and  $(n-r)$  elements are redundant.

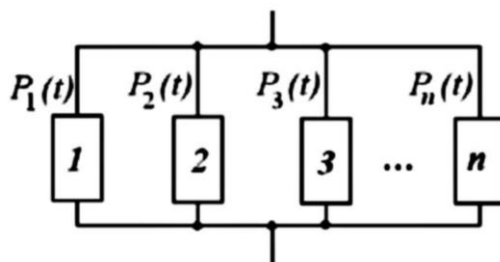


Figure 3 - Block diagram of the reliability of a system with parallel connection of elements  
Рисунок 3 - Структурная схема надежности системы с параллельным соединением элементов

As long as the number of backup elements exceeds the number of failed elements, the system is functional. Thus, the failure condition has the form:

$$m - 1 = n - r \quad (14)$$

The probability of system failure is determined as the probability of matching the failure of elements  $m - 1 = n - r$  during the estimated time [1].

$$Q_i(t) \text{ and } P_i(t) \left[ P_i(t) + Q_i(t) \right] = 1$$

$$\prod_{i=1}^n \left[ P_i(t) + Q_i(t) \right] \left[ P_1(t) \cdot P_2(t) \cdot \dots \cdot P_n(t) \right] + \left[ Q_1(t) \cdot P_2(t) \cdot \dots \cdot P_n(t) \right] + \left[ Q_1(t) \cdot Q_2(t) \cdot \dots \cdot Q_n(t) \right] \quad (15)$$

By selecting terms with the value  $Q_i(t)$  from this sum, you can form an expression for the probability of failure from them. Based on the product of the binomial  $n$ , you can get by constructing the sum of the probability product  $n$

If the elements are equally reliable, we get:

$$Q(t) = \sum_{k=r}^n C_n^k P^{n-k}(t) Q^k(t) \quad (16)$$

$$P(t) = \sum_{k=r}^n C_n^k P^k(t) Q^{n-k}(t) \quad (17)$$

$$\text{where } C_n^k = \frac{n!}{(n-k)!k!}.$$

The condition of mutual independence of element failures is fulfilled if there is no significant overload of elements when the number of elements in operation changes. Thus, a system with a parallel connection of elements is a redundant system, i.e. a failure of one or more elements does not cause a failure of the entire system.

In reliability theory, there are two types of redundancy:

Redundancy is called permanent if all elements are constantly in operation, and the system does not fail until a certain number of them fail.

If the backup elements are enabled only after the failed elements are automatically disabled, a replacement backup occurs.

In the power industry, substitution redundancy is performed by numerous AVR devices ("cold" reserve), permanent-by rotating and hidden reserve of generators, transformers and electric motors ("hot" reserve) [2].

As a quantitative indicator of efficiency of functioning of the electricity system taking the ratio of real output to the ideal effect, i.e. the ratio of expected released power consumer in the perfect performance of the system. Quantitative assessment of the efficiency of operation is one of the final results of all calculations of the reliability of power supply systems. It is obvious that a quantitative assessment of the system's effectiveness should be based on quantitative

indicators of its reliability. At this stage of scientific knowledge, i.e. at the stage of formation, the problem of reliability in energy and power supply is relevant.

#### REFERENCES

- [1] Shemetov A. N. Reliability of power supply: textbook for students of specialty 140211 "Power Supply". Magnitogorsk: Moscow state technical University named after G. I. Nosov, 2006. [http://magtu-epp.narod.ru/literature/Nadejnost\\_el\\_snab.pdf](http://magtu-epp.narod.ru/literature/Nadejnost_el_snab.pdf)
- [2] Volkov N. G. Reliability of functioning of power supply systems. Textbook. - Tomsk: TPU publishing House, 2005. – 157 с. [https://portal.tpu.ru/SHARED/i/IOM/liter/Tab/M\\_Volkov\\_Nad\\_fun\\_sist\\_el\\_snab\\_2005.pdf](https://portal.tpu.ru/SHARED/i/IOM/liter/Tab/M_Volkov_Nad_fun_sist_el_snab_2005.pdf)
- [3] Safonov V. I. Reliability of power supply systems. Textbook. - Lonzinger-Chelyabinsk: SUSU Publishing center, 2014. – 90 с. <https://ses.susu.ru/wp-content/uploads/2016/03/Сафонов-В.И.-Надежность-электроснабжения.-Конспект-лекций.pdf>
- [4] Basmanov V. G., Kholmanskikh V. M. Connection of reliability indicators of electric equipment of cranes with their productivity at the enterprises of the timber industry complex // Modern problems of science and education. – 2014. – № 3; <https://fundamental-research.ru/ru/article/view?id=38201>
- [5] Kiselov V. K. Bases of calculation of reliability of power systems: textbook. manual / guvpo "Ivanovo state power engineering University named after V. I. Lenin". – Ivanovo, 2012. – 80 с. [http://xn--b1ajwv.xn--p1ai/files/l\\_ne.pdf](http://xn--b1ajwv.xn--p1ai/files/l_ne.pdf)

#### ЛИТЕРАТУРА

- [1] Шеметов А. Н. Надежность электроснабжения: учебное пособие для студентов специальности 140211 "Электроснабжение". - Магнитогорск: Московский государственный технический университет им. Г. И. Носова, 2006. [http://magtu-epp.narod.ru/literature/Nadejnost\\_el\\_snab.pdf](http://magtu-epp.narod.ru/literature/Nadejnost_el_snab.pdf)
- [2] Волков Н. Г. Надежность функционирования систем электроснабжения. Учебник. - Томск: Изд – во ТПУ, 2005. – 157 с. [https://portal.tpu.ru/SHARED/i/IOM/liter/Tab/M\\_Volkov\\_Nad\\_fun\\_sist\\_el\\_snab\\_2005.pdf](https://portal.tpu.ru/SHARED/i/IOM/liter/Tab/M_Volkov_Nad_fun_sist_el_snab_2005.pdf)
- [3] Сафонов В. И. Надежность систем электроснабжения. Учебник. - Лонцингер-Челябинск: Издательский центр ЮУРГУ, 2014. – 90 с. <https://ses.susu.ru/wp-content/uploads/2016/03/Сафонов-В.И.-Надежность-электроснабжения.-Конспект-лекций.pdf>
- [4] Басманов В. Г., Холманских В. М. Связь показателей надежности электрооборудования кранов с их производительностью на предприятиях лесопромышленного комплекса // Современные проблемы науки и образования. – 2014. – № 3; <https://fundamental-research.ru/ru/article/view?id=38201>
- [5] Киселев В. К. Основы расчета надежности энергосистем: учебное пособие. учебное пособие / ГУВПО "Ивановский государственный энергетический университет имени В. И. Ленина". – Иваново, 2012. – 80 с. [http://xn--b1ajwv.xn--p1ai/files/l\\_ne.pdf](http://xn--b1ajwv.xn--p1ai/files/l_ne.pdf)

#### МАТЕМАТИЧЕСКИЕ МОДЕЛИ НАДЕЖНОСТИ СИСТЕМ ЭЛЕКТРОСНАБЖЕНИЯ

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**Аннотация.** Обеспечение надежности является одним из важнейших вопросов при создании и эксплуатации любой технической системы. Это особенно важно для сложных систем, таких как системы электроснабжения с внутренними и внешними соединениями, состоящими из множества элементов. Задача обеспечения надежности систем электроснабжения включает в себя целый комплекс технических, экономических и организационных мероприятий, направленных на снижение ущерба, причиняемого нарушениями нормального режима работы потребителей электрической энергии. Для получения информации о надежности элементов систем электроснабжения необходимо изучить достаточно большое количество идентичных элементов в течение длительного периода времени. В реальных условиях промышленного предприятия это создает определенные трудности, поэтому возможно проведение ускоренной проверки надежности с использованием математического моделирования [3].

В статье рассматриваются некоторые пути решения основной задачи теории надежности - построения математической модели и получения критериев выбора оптимальных вариантов

систем электроснабжения. В данной работе представлены основные материалы для расчета надежности систем электроснабжения с использованием математического аппарата теории вероятностей.

**Ключевые слова:** надежность, электроснабжение, математическая модель, параметры, резервирование системы, отказ, теория вероятностей.

### ЭЛЕКТРМЕН ЖАБДЫҚТАУ ЖҮЙЕЛЕРІ СЕНІМДІЛІГІНІҢ МАТЕМАТИКАЛЫҚ МОДЕЛЬДЕРІ

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**Аңдатпа.** Сенімділікті қамтамасыз ету кез келген техникалық жүйені құру және пайдалану кезіндегі маңызды мәселелердің бірі болып табылады. Бұл көптеген элементтерден тұратын ішкі және сыртқы қосылыстары бар электрмен жабдықтау жүйелері сияқты күрделі жүйелер үшін өте маңызды. Электрмен жабдықтау жүйелерінің сенімділігін қамтамасыз ету міндеті электр энергиясын тұтынушылардың қалыпты жұмыс режимін бұзудан болатын зиянды азайтуға бағытталған техникалық, экономикалық және ұйымдастырушылық шаралардың тұтас кешенін қамтиды. Электрмен жабдықтау жүйелері элементтерінің сенімділігі туралы ақпарат алу үшін ұзақ уақыт бойына бірдей элементтердің жеткілікті үлкен санын зерттеу қажет. Өнеркәсіптік кәсіпорынның нақты жағдайында бұл белгілі бір қиындықтар туғызады, сондықтан математикалық модельдеуді қолдана отырып, сенімділікті жедел тексеруге болады [3].

Мақалада сенімділік теориясының негізгі мәселесін шешудің кейбір жолдары: математикалық модель құру және электрмен жабдықтау жүйелерінің оңтайлы нұсқаларын таңдау критерийлерін алу қарастырылады. Бұл жұмыста ықтималдықтар теориясының математикалық аппаратын қолдана отырып, электрмен жабдықтау жүйелерінің сенімділігін есептеуге арналған негізгі материалдар ұсынылған.

**Түйінді сөздер:** сенімділік, электрмен жабдықтау, математикалық модель, параметрлер, жүйені резервтеу, істен шығу, ықтималдық теориясы.

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### WAYS TO IMPROVE THE TECHNOLOGY AND EQUIPMENT OF ELECTRIC CENTIFUGAL PUMPS

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**Abstract.** In this article, a method of field development using gas separators is developed based on reducing the harmful effects of gas on dual electric centrifugal pumps. In the absence of high-performance serial equipment on the example of the 25568/899 well, the "Double" installation showed conditions that allow using a well with a good flow rate. It was concluded that the technology of "dual" electric centrifugal pumps was applied.

The content of free gas at the reception and inside the gas separator is usually large, so its flow organs inevitably show developed flows in artificial cavitation modes with the formation of large gas caverns (supercaverns). During supercavitation flow around the blade profile by the gas-liquid mixture flow, there is a significant enlargement of the gas bubbles that break away from the blade, that is, there is a local phase separation in the flow.